

For full credit, show all the steps clearly in a logical and legible manner. Right answer without right steps shown will NOT get you full credit.

Problem 1 (30 points): Consider a two-dimensional flow (in the oxy plane) with velocity field given by

$$u = 2ax + 0.3y + t \quad \text{and} \quad v = bx - 0.2y$$

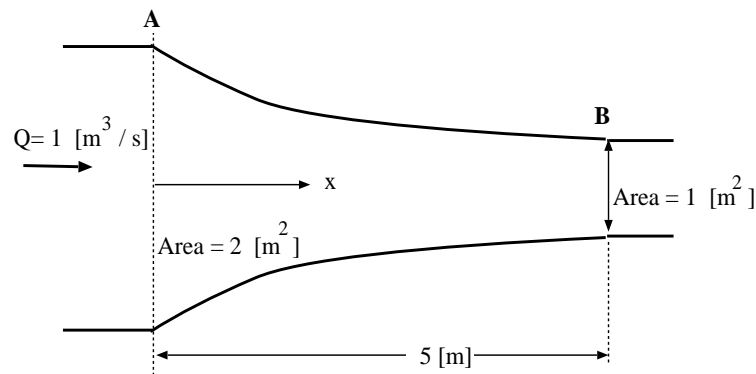
where a and b are unknown constants, x and y are spatial coordinates in the unit of [m] and t denotes time in the unit of [s]. The unit of u and v is [m/s].

Knowing that the fluid is incompressible and that the flow is irrotational, determine

- (i) the constants a and b ...(5+5 = 10 points)
 (ii) x-component of fluid acceleration a_x at $x = 1$ [m], $y = 1$ [m] and $t = 3$ [s]. ...(10 points)
 (iii) y-component of fluid acceleration a_y at $x = 1$ [m], $y = 1$ [m] and $t = 3$ [s]. ...(10 points)

Problem 2 (30 points): Consider an one-dimensional horizontal flow of an incompressible fluid along a transition, as shown in the figure below. The section area at the left end A of the transition is $2 \text{ [m}^2\text{]}$ and at the right end B is $1 \text{ [m}^2\text{]}$. The length of the transition is 5 [m] . The geometry of the section is such that the velocity varies linearly between A and B. The flow is from left to right with volume flow rate $Q = 1 \text{ [m}^3\text{/s]}$. Assuming that velocity is uniform across each cross section, determine

- (i) velocity at $x = 0$ [m], 5 [m] and 3 [m] ...(5+5+5 = 15 points)
 (ii) flow acceleration a_x at $x = 4$ [m] ...(15 points)



Problem 3 (40 points): Consider vertical oscillation of an incompressible and inviscid fluid column in a tube, as shown in the figure below. The fluid column oscillates up and down sinusoidally with velocity

$$w = 0.1 \cos(2t) \quad [\text{m/s}]$$

The tube is open to the atmosphere at the top and the mass of the piston at the bottom is negligible. The section-area of the tube is $0.1 \text{ [m}^2\text{]}$ and the height of the fluid column is 1 [m] . The fluid density $\rho = 1000 \text{ [kg/m}^3\text{]}$ and the acceleration of gravity $g = 9.8 \text{ [m/s}^2\text{]}$.

Assuming that the flow is one dimensional (in the vertical direction), that all fluid particles have the same velocity at any instant of time and that the free surface (interface between the fluid and atmosphere) remains flat, determine an expression for the force applied on the bottom piston.

Note: You may either use the Newton's II Law or the Euler's equation for the solution of this problem.

