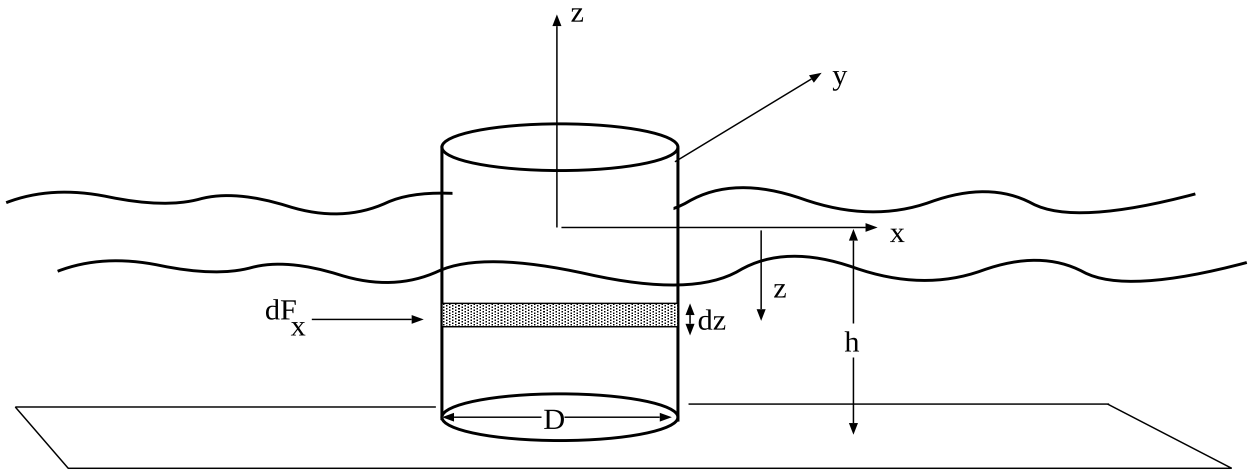
EOC4422 **Ocean Wave Mechanics** **Fall 2006**

Term Project (Individual effort; 15% of the grade) Due by 06 December 2006

WAVE FORCE



1. Froude-Krylov Force

Consider a vertical fixed piling of diameter D in water of depth h. Let the height and length of incident wave, progressing along the x direction, be denoted as H and L, respectively. Assuming that the body diameter D is very small compared to the incident wave length, one can determine the wave exciting force by integrating the pressure field of the incident wave only. The assumption, which neglects wave diffraction by the body, is known as the Froude-Krylov hypothesis.

The wave exciting force on the body, per Froude-Krylov hypothesis, is then given by

$$\vec{F} = -\int_S p^i_{dyn}\ \hat{n}\ dS$$

where $\hat{n}$ denotes outward normal vector on the surface of the cylinder and p^i_{dyn} the dynamic pressure of the incident wave. Using the Gauss theorem (for a scalar) the above surface integral may be transformed into a volume integral which perhaps is easier, from computational voewpoint, to evaluate:

$$\vec{F} = -\int_\forall \nabla p^i_{dyn}\ d\forall$$

where $\forall$ denotes the volume of the cylinder. The x component of the wave force is thus

$$F_x = -\int_\forall \frac{\partial p^i_{dyn}}{\partial x}\ d\forall$$

The dynamic pressure field of a small-amplitude progressive wave is given by (refer to Chapter 3)

$$p^i_{dyn} = \rho g \frac{H}{2} \frac{\text{COSH}k(z+h)}{\text{COSH}kh}\ \sin(kx-\sigma t)$$

and therefore

$$\frac{\partial p^i_{dyn}}{\partial x} = \rho g k \frac{H}{2} \frac{\text{COSH}k(z+h)}{\text{COSH}kh}\ \cos(kx-\sigma t)$$

i. X-Component of the Wave Exciting Force, F_x

The x-component of the Froude-Krylov force is then

$$F_x = - \int_{\forall} \rho g k \frac{H}{2} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \cos(kx - \sigma t) \; d\forall$$

Assuming the cylinder to be slender, ie. $D/L << 1$, one can evaluate the above integral by setting $x = 0$; ie.,

$$F_x \approx - \int_{\forall} \rho g k \frac{H}{2} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \cos(\sigma t) \; d\forall$$

Above can now be integrated with respect to z as,

$$\begin{aligned} F_x &= - \int_{z=-h}^{z=0} \rho g k \frac{H}{2} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \cos(\sigma t) \; \pi \frac{D^2}{4} \; dz \\ &= -\rho g k \frac{H}{2} \pi \frac{D^2}{4} \frac{\cos(\sigma t)}{\text{COSH} kh} \int_{z=-h}^{z=0} \text{COSH} k(z+h) \; dz \\ &= -\rho g k \frac{H}{2} \pi \frac{D^2}{4} \frac{\cos(\sigma t)}{\text{COSH} kh} \left(\frac{\text{SINH} k(z+h)}{k} \right)_{z=-h}^{z=0} \\ &= -\rho g \frac{H}{2} \pi \frac{D^2}{4} \; \text{TANH} kh \cos(\sigma t) \end{aligned}$$

The amplitude of the force in the direction of wave propagation is therefore

$$|F_x| = \rho g \frac{H}{2} \pi \frac{D^2}{4} \; \text{TANH} kh \tag{1}$$

ii. Vertical Distribution of the Wave Exciting Force per Unit Length, dF_x/dz

In the present case, the distribution of the Froude-Krylov force F_x per unit length along the vertical direction, ie. dF_x/dz can also be easily determined. Again assuming $D << L$ (ie., cylinder diameter smaller than the incident wave length), one can write the differential force dF_x on an element of height dz of the cylinder as

$$dF_x = - \rho g k \frac{H}{2} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \cos(\sigma t) \; \pi \frac{D^2}{4} dz$$

Therefore, the vertical distribution of wave force per unit length dF_x/dz can be written as

$$\frac{dF_x}{dz} = - \rho g k \frac{H}{2} \; \pi \frac{D^2}{4} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \cos(\sigma t)$$

and its amplitude as

$$|\frac{dF_x}{dz}| = \rho g k \frac{H}{2} \; \pi \frac{D^2}{4} \frac{\text{COSH} k(z+h)}{\text{COSH} kh} \tag{2}$$

2. Inertia and Drag Force - Morrison Equation Method

Next let us consider the Morrison-equation method for determining both inertia and drag components of in-line hydrodynamic force. Recall, per Morrison equation method, the in-line hydrodynamic force on an element of differential height dz (of the cylinder shown in the Figure on page 1) can be written as

$$\begin{aligned} dF_x &= dF_D + dF_I \\ &= C_D \frac{\rho}{2} u|u| dA_p + C_I \rho dotud\forall \\ &= C_D \frac{\rho}{2} u|u| D \ dz + C_I \rho dotu \ \frac{\pi D^2}{4} \ dz \end{aligned}$$

From linear-wave theory,

$$\begin{aligned} u &= \frac{H}{2} \frac{gk}{\sigma} \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \sin(kx - \sigma t) \\ \dot{u} &\approx \frac{\partial u}{\partial t} \ \text{for small-amplitude wave} \\ &= -\frac{H}{2} gk \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \cos(kx - \sigma t) \end{aligned}$$

and therefore

$$\begin{aligned} dF_D &= C_D \frac{\rho}{2} u|u| D \ dz \\ &= C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma} \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \right)^2 \sin(kx - \sigma t) |\sin(kx - \sigma t)| \ dz \end{aligned}$$

and

$$\begin{aligned} dF_I &= C_I \rho \dot{u} \ \frac{\pi D^2}{4} \ dz \\ &= -C_I \ \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \cos(kx - \sigma t) \ dz \end{aligned}$$

Assuming $x = 0$ (that of the axis), the distribution of force per unit length along the vertical direction can be written as

$$\frac{dF_D}{dz} = C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma} \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \right)^2 \sin(-\sigma t) |\sin(-\sigma t)|$$

and

$$\frac{dF_I}{dz} = -C_I \ \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \cos(\sigma t)$$

Integrating along the z direction, one can determine the total drag and inertia components of the force as

$$F_D = \int_{z=-h}^{z=0} C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma} \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \right)^2 \sin(-\sigma t) |\sin(-\sigma t)| \ dz$$

$$F_I = \int_{z=-h}^{z=0} -C_I \ \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{\text{COSH}k(z+h)}{\text{COSH}kh} \cos(\sigma t) \ dz$$

If one further assumes C_D and C_I to be independent of z, the above expressions for drag and inertia components of wave force become

$$\begin{aligned}
F_D &= C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma \, \mathrm{COSH} kh} \right)^2 \sin(-\sigma t) |\sin(-\sigma t)| \int_{z=-h}^{z=0} \mathrm{COSH}^2 k(z+h) \; dz \\
&= C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma \, \mathrm{COSH} kh} \right)^2 \sin(-\sigma t) |\sin(-\sigma t)| \int_{z=-h}^{z=0} \frac{1}{2} [\mathrm{COSH} 2k(z+h) + 1] \; dz \\
&= C_D \frac{\rho}{2} D \left(\frac{H}{2} \frac{gk}{\sigma \, \mathrm{COSH} kh} \right)^2 \sin(-\sigma t) |\sin(-\sigma t)| \; \frac{1}{2} \; [\frac{\mathrm{SINH} \; 2kh}{2k} + h] \quad \text{(please check for typos).}
\end{aligned}$$

and

$$\begin{aligned}
F_I &= \int_{z=-h}^{z=0} -C_I \; \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{\mathrm{COSH} k(z+h)}{\mathrm{COSH} kh} \cos(\sigma t) \; dz \\
&= -C_I \; \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{1}{\mathrm{COSH} kh} \cos(\sigma t) \int_{z=-h}^{z=0} \mathrm{COSH} k(z+h) \; dz \\
&= -C_I \; \rho \frac{\pi D^2}{4} \frac{H}{2} gk \frac{1}{\mathrm{COSH} kh} \cos(\sigma t) \; \frac{\mathrm{SINH} \; kh}{k} \qquad \text{(please check for typos).}
\end{aligned}$$

3. Project Work

Determine the the vertical distribution of wave force per unit length dF_x/dz and total wave force F_x using the above Froude-Krylow and Morrison Equation methods for a vertical piling in waves for the following range of parameter values:

Item	Particulars
Diameter, D	1 [m], 2 [m]
Wave Depth, h	5 [m], 100 [m]
Wave Length, L	10 [m], 100 [m]
Wave Height, H	1 [m], 2 [m]
Drag Coefficient, C_D	1.2
Inertia Coefficient, C_I	2.1

For reporting, consider about 5 to 10 cases spanning broad range of above parameters. The report must contain the results and a scholarly discussion of the results. The results may be presented in the form of graphs and tables. The report will thus contain

1. This assignment write-up
2. Your results and discussion of results

which must be turned in by **06 December 2006** at SeaTech. You may want to keep a copy of the report, in case the report is lost or misplaced and I seek another copy and also as you may want to refer to it in the future.