EOC4422 OCEAN WAVE MECHANICS FALL 2007

A Note on the Sign Convention for Velocity Potential

Often Used Convention: $\vec{u} = +\nabla\phi$

For an irrotational ($\nabla \times \vec{u} = 0$), $\vec{u} = +\nabla\phi$.

In addition if the flow is incompressible, $\nabla \cdot \vec{u} = 0 \rightarrow \nabla \cdot (\nabla\phi) = 0 \rightarrow \nabla^2\phi = 0$ which is called the Laplace equation.

Substituting $\vec{u} = +\nabla\phi$ in the Euler's equation and integrating in space, one obtains (for an unsteady flow)

$$\rho\frac{\partial\phi}{\partial t} + \frac{\rho}{2}|\nabla\phi|^2 + \rho g z + p = 0$$

Or,

$$p = -\rho g z - \rho\frac{\partial\phi}{\partial t} - \frac{\rho}{2}|\nabla\phi|^2$$

which is called the Euler's integral (or) the unsteady Bernoulli's equation.

Convention Used in the Text Book: $\vec{u} = -\nabla\phi$

Again for an irrotational ($\nabla \times \vec{u} = 0$), $\vec{u} = -\nabla\phi$. In other words, it does not matter whether $\vec{u} = +\nabla\phi$ or $\vec{u} = -\nabla\phi$, curl of both is identically zero; ie., $\nabla \times \pm\nabla\phi \equiv 0$. The text follows the $\vec{u} = -\nabla\phi$. One must be careful to make note of this in subsequent equations.

For example, in addition if the flow is incompressible, $\nabla \cdot \vec{u} = 0 \rightarrow \nabla \cdot (-\nabla\phi) = 0 \rightarrow -\nabla^2\phi = 0$. Since right hand side is zero, this is same the original Laplace equation: $\nabla^2\phi = 0$.

Substituting $\vec{u} = -\nabla\phi$ in the Euler's equation and integrating in space, one obtains (for an unsteady flow)

$$-\rho\frac{\partial\phi}{\partial t} + \frac{\rho}{2}|-\nabla\phi|^2 + \rho g z + p = 0$$

Since $|-\nabla\phi| \equiv |\nabla\phi|$, above can be written as

$$-\rho\frac{\partial\phi}{\partial t} + \frac{\rho}{2}|\nabla\phi|^2 + \rho g z + p = 0$$

Or as,

$$p = -\rho g z + \rho\frac{\partial\phi}{\partial t} - \frac{\rho}{2}|\nabla\phi|^2$$

which is called the Euler's integral (or) the unsteady Bernoulli's equation. Note that the second term on the right has a + sign.

Henceforth in this course, we will the text book convention and denote $\vec{u} = -\nabla\phi$. One must however be aware of the other convention that is commonly found in the literature and that is the purpose of this brief note.
