13. WAVE FORCE ON A BODY: FROUDE-KRYLOV METHOD

Next,we shall discuss a method which can be used to determine inviscid force on a body in waves. As illustrated in Fig 13-1, the presence of an object will alter the ambient (ie. incident) waves, and this process is known as wave scattering or diffraction If the body size is small, it is intuitively obvious that the wave scattering by the body (ie. the body's effect on the ambient waves) will be negligible. By small body size we precisely mean that the body length D is small compared to the wavelength L; ie., D/L << 1. Throw, for example, a pop corn in the ocean. The ocean waves will not be affected by the pop corn, but the pop-corn's dynamics will be affected by the ocean waves. For cases in which wave diffraction is negligible, one can simply integrate the pressure of the incident wave field to determine the wave forces. This was first hypothesized by Froude and Krylov, and therefore the force of the incident wave pressure is also known as the Froude-Krylov force. Curiously, in the text book the name Froude-Krylov is not mentioned even though it is so known in the ocean engineering / naval architecture community. If the body size is large or comparable to the incident wave length (eg., a huge ship or offshore structure in waves), the effect of wave diffraction will become pronounced and the hydrodynamic force will be due to both the incident- and diffracted-wave pressure fields. Also note that if Froude-Krylov force is based on the potential-flow solution of the incident wave pressure, it cannot predict the drag force. As you may recall from earlier discussion, the drag/inertia force ratio is proportional to H/D. The classical Froude-Krylov method based on potential-flow solution is therefore applicable for cases in which both D/L and H/D ratios are << 1.

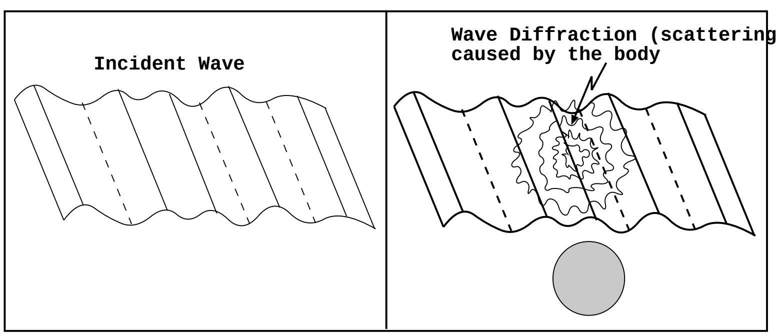

Fig 13-1. Scattering of waves by a body

Application: Froude-Krylov Force on a Vertical Piling. Consider a vertical piling of circular section with diameter D in a *long-crested* wave progressing in the x direction with wave height H, water depth h, frequency σ and wavelength L, as depicted in Fig 13-2. Let $D/L << 1$, ie, diffraction of waves by the piling is negligible. From linear wave solutions, we know that the dynamic pressure of the incident wave is given by

$$p = \rho g \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \sin(kx - \sigma t)$$

Note that this relation was obtained earlier without considering any boundary conditions for the presence of the piling. If the body is large, the pressure field will be affected by the piling boundary via wave diffraction. Here, as the piling is small we assume the effect of the piling on the wave to be negligible.

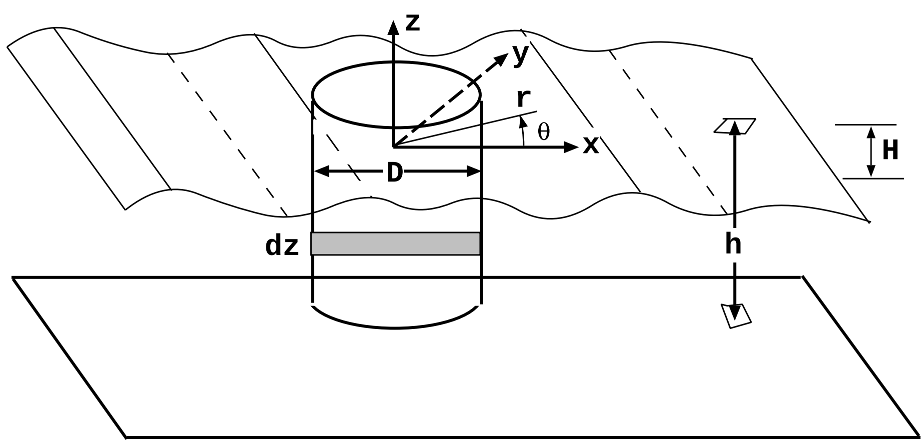

Fig 13-2. A vertical piling in waves

Keep in mind that the Froude-Krylov hypothesis does not involve water depth h, which can be large, comparable or small compared to the wave length. Only the diameter of the piling D is assumed to be small.

Now consider a slice of the piling of differential height dz, as shown in the figure. The Froude-Krylov force on the slice is given by

$$d\vec{F} = -\int p\ \hat{n}\ dS$$

where $\hat{n}$ is a unit normal vector on the body surface and pointing outward of the body. As pressure force on the body is normal and in the direction opposite of $\hat{n}$, a negative sign is introduced in the integral. From the corollary of the Gauss theorem,

$$\int_S a\ \hat{n}\ dS \equiv \int_\forall \nabla p\ d\forall \quad \text{(where a is a scalar function)},$$

the above equation for the force can be written as

$$d\vec{F} = -\int \nabla p\ \ d\forall$$

As the cylinder diameter is small, the pressure-gradient term ∇p may be approximated to be that at the center (ie., at x=y=0). Replacing ∇p by ∇p_o where p_o corresponds to the pressure on the axis of the cylinder (x=y=0), the above equation reduces to

$$d\vec{F} = -\int \nabla p_o\ \ d\forall = -\nabla p_o \int\ d\forall = -\nabla p_o\ \delta\forall$$

where $\delta\forall$ denotes the volume of the slice. Above is a vector relation, and the x-component of the force on the slice is therefore

$$dF_x = -\left(\frac{\partial p}{\partial x}\right)_o\ \ \delta\forall$$

Using the expression for the dynamic pressure in a long-crested progressive wave given by

$$p = \rho g \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \sin(kx - \sigma t)$$

and volume of the slice $\delta\forall = \frac{1}{4}\pi D^2 \; dz$, we obtain for the force on the slice

$$\begin{aligned} dF_x &= -\left(\rho g k \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \cos(kx - \sigma t)\right)_{x=y=0} \frac{\pi D^2}{4} \; dz \\ &= -\frac{\pi D^2}{4} \rho g k \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \cos(\sigma t) \; dz \end{aligned}$$

By integrating from $z = -h$ (sea bottom) to $z = 0$, one can determine the total wave exciting force F_x:

$$\begin{aligned} F_x &= \int_{z=-h}^{z=0} dF_x \\ &= -\frac{\pi D^2}{4} \rho g k \frac{H}{2} \frac{1}{\cosh kh} \cos(\sigma t) \int_{z=-h}^{z=0} \cosh k(z+h) \; dz \\ &= -\frac{\pi D^2}{4} \rho g k \frac{H}{2} \frac{1}{\cosh kh} \cos(\sigma t) \left(\frac{1}{k} \sinh k(z+h)\right)_{z=-h}^{z=0} \\ &= -\frac{\pi D^2}{4} \rho g \frac{H}{2} \frac{1}{\cosh kh} \cos(\sigma t) \sinh kh \\ &= -\rho g \frac{H}{2} \frac{\pi D^2}{4} \mathrm{Tanh}(kh) \cos(\sigma t) \end{aligned}$$

(How difficult is it to use this final formula to compute the wave exciting force on the vertical piling?!).

Wave Overturning Moment. In practice, it is also necessary to know the wave force bending moment about the bottom for the design of the piling foundation. The moment due to the force dF_x about the bottome (z=-h) is given by

$$dM = (z+h) \; dF_x$$

and therefore the total moment by

$$M = \int_{z=-h}^{z=0} (z+h) \; dF_x$$

where, from above, dF_x is given by

$$dF_x = -\frac{\pi D^2}{4} \rho g k \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \cos(\sigma t) \; dz$$

and therefore

$$\begin{aligned} M &= -\int_{z=-h}^{z=0} (z+h) \frac{\pi D^2}{4} \rho g k \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \cos(\sigma t) \; dz \\ &= -\rho g k \frac{H}{2} \frac{\pi D^2}{4} \frac{1}{\cosh kh} \cos(\sigma t) \int_{z=-h}^{z=0} (z+h) \cosh k(z+h) \; dz \end{aligned}$$

(Evaluation of the integral is left to the reader as an exercise!)
