## 12b. WAVE FORCE ON A BODY: MORRISON EQUATION METHOD

**Keulegan-Carpenter (K-C) Number.** Earlier we showed that the ratio of drag and inertia components of the wave force is governed by

$$O\left(\frac{|F_D|}{|F_I|}\right) = \frac{H}{D}\frac{1}{\text{Tanh}kh}$$

One can show that this ratio is also related to the more familiar parameter in fluid mechanics; namely, Keulegan-Carpenter (K-C) number defined for oscillating flows about a fixed body. Consider an oscillating flow with amplitude of velocity $U_{max}$, period of oscillation $T$ about a body of size $D$. The K-C number is defined as [1]

$$\text{K-C} \equiv \frac{U_{max}T}{D}$$

In the case of a progressive wave, the horizontal component of velocity is given by

$$u = \frac{H}{2}\frac{gk}{\sigma}\;\frac{\cosh k(z+h)}{\cosh kh}\;\sin(kx-\sigma t)$$

Near the surface (ie., $kz << 1$), the amplitude of the horizontal velocity is

$$U_{max} = \frac{H}{2}\frac{gk}{\sigma}$$

Using the dispersion relation $\sigma^2 = gk\ \text{Tanh}kh$, the above equation can be written as

$$U_{max} = \frac{H}{2}\frac{\sigma}{\text{Tanh}kh}$$

The K-C number for this flow is then

$$\text{K-C} \equiv \frac{U_{max}T}{D} = \frac{H}{2}\frac{\sigma}{\text{Tanh}kh}\;\frac{T}{D}$$

As $T = 2\pi/\sigma$, the above equation becomes

$$\text{K-C} = \frac{H}{2}\frac{\sigma}{\text{Tanh}kh}\;\frac{2\pi}{\sigma}\frac{1}{D} = \pi\left(\frac{H}{D}\frac{1}{\text{Tanh}kh}\right)$$

[1] Do not mix the K-C number with the Strouhal number associated with vortex shedding frequency and defined as St = nD/U (refer to Fluids I text).

We had shown earlier (see Eqn.1 on p.54) that the order of magnitude of drag-to-inertia force ratio is given by

$$O\left(\frac{F_D}{F_I}\right) = \left(\frac{H}{D}\frac{1}{\text{Tanh}kh}\right)$$

By comparing the two relations above, we note that the ratio of drag and inertia components of the wave force is also related to the K-C number as

$$O\left(\frac{F_D}{F_I}\right) = \frac{H}{D}\frac{1}{\text{Tanh}kh} = \pi \text{ K-C}$$

The $\pi$ factor may be neglected in the order of magnitude ratio of the forces. The above relation means that higher the K-C number the more significant will be the drag component of the force over the inertia component and *vice versa*. Hypothetical flow structure around a cylinder at low and high K-C numbers are shown in the figure below. Intuitively, at low KC number the flow is not highly vortical around the cylinder implying that the drag force will not be significant compared to the inertia force. At high KC number, flow is expected to be highly vortical around the cylinder and hence contributing more to the drag component of the hydrodynamic force.

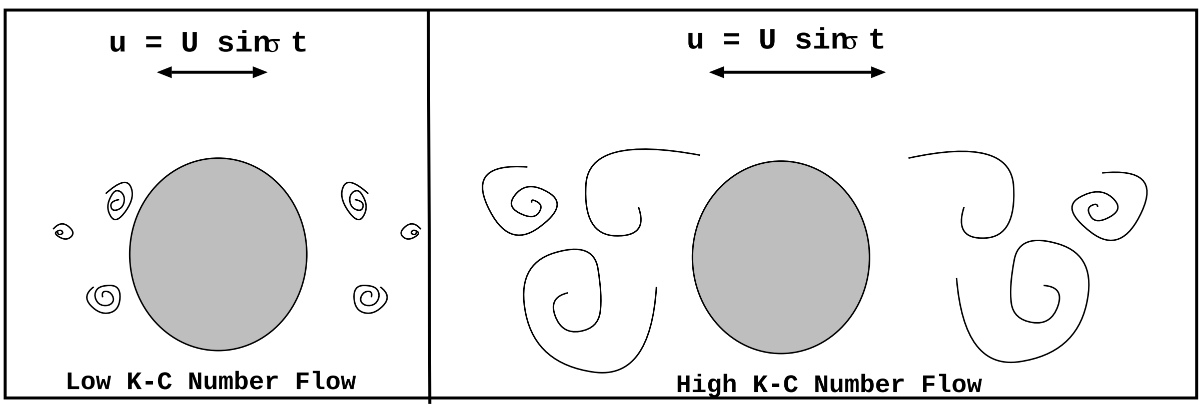


Fig 12-1. Flow Cartoon of oscillating Flow past a cylinder at low and high KC numbers

Recent research of the instructor (P. Ananthakrishnan), among others, have shown that the drag and inertia components of hydrodynamic force is governed not only by the KC number (or H/D Tanh[kh] ) ratio but also by other quantities such as flow convection, wave height/wave length ratio (which is a measure of wave nonlinearity) and fluid particle trajectories. If you are interested in pursuing such research on wave-body and wave-vehicle interactions for graduate degree, talk to the instructor (P. Ananthakrishnan).