10. DRAG FORCE ON A CYLINDER: Potential and Viscous Results

Having determined the velocity potential of unsteady flow about a stationary cylinder as

$$\phi(r,\theta,t) = U(t)\{r + \frac{a^2}{r}\}\ \cos\theta,$$

let us now compute the dynamic pressure on the cylinder, and then by integration, the hydrodynamic force acting on the cylinder.

By Euler's integral, the expression for dynamic pressure is given by

$$p = \rho\frac{\partial\phi}{\partial t} - \frac{\rho}{2}|\nabla\phi|^2$$

where, for the flow past the cylinder,

$$\frac{\partial\phi}{\partial t} = \frac{dU}{dt}\{r + \frac{a^2}{r}\}\ \cos\theta$$

and

$$|\nabla\phi|^2 = u_r^2 + u_\theta^2 = (-\frac{\partial\phi}{\partial r})^2 + (-\frac{1}{r}\frac{\partial\phi}{\partial\theta})^2 = \{-U(1-\frac{a^2}{r^2})\cos\theta\}^2 + \{U(1+\frac{a^2}{r^2})\sin\theta\}^2$$

The pressure far upstream ($r = l,\ \theta = 0$, where $r >> l$) is chosen as reference pressure, which from the Euler's integral and above relations can be written as:

$$p_{(r=l,\ \theta=0)} \equiv p_o = \rho\frac{dU}{dt}\{r + \frac{a^2}{r}\}\ \cos\theta|_{(r=l,\theta=0)} - \frac{\rho}{2}\{-U(1-\frac{a^2}{r^2})\cos\theta\}^2|_{(r=l,\theta=0)}$$
$$-\frac{\rho}{2}\{U(1+\frac{a^2}{r^2})\sin\theta\}^2|_{(r=l,\theta=0)}$$

For $l >> a$, the above expression for pressure in the far-field ($r = l, \theta = 0$) becomes,

$$p_o = \rho l\frac{dU}{dt} - \frac{\rho}{2}U^2$$

On the surface of the cylinder ($r = a$), the expression for dynamic pressure is given by

$$\begin{aligned} p_{(r=a,\theta)} &= \rho\frac{dU}{dt}\{r + \frac{a^2}{r}\}\ \cos\theta|_{(r=a,\theta)} - \frac{\rho}{2}\{-U(1-\frac{a^2}{r^2})\cos\theta\}^2|_{(r=a,\theta)} \\ &\quad -\frac{\rho}{2}\{U(1+\frac{a^2}{r^2})\sin\theta\}^2|_{(r=a,\theta)} \\ &= 2\rho a\frac{dU}{dt}\ \cos\theta - \frac{\rho}{2}(4U^2\ \sin^2\theta) \end{aligned}$$

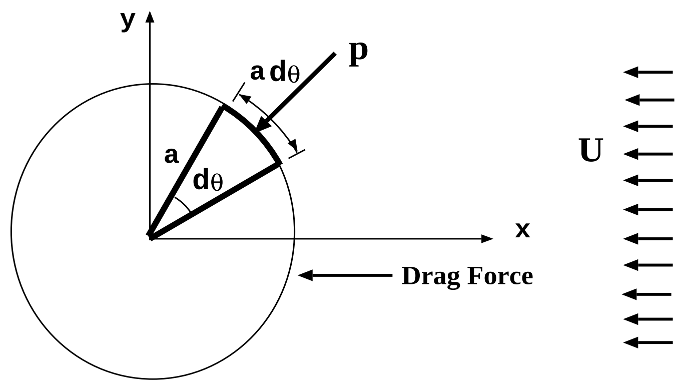

Fig 10-1. Flow past a cylinder

The dynamic gage pressure on the cylinder is therefore

$$p_{cyl} \equiv p_{(r=a,\theta)} - p_O = \frac{\rho}{2}U^2(1-4\sin^2\theta) + \rho\frac{dU}{dt}(2a\ \cos\theta - l)$$

The dynamic pressure (gage) on the cylinder consists of two sets of terms, one proporional to U^2 which is referred to as the steady pressure term, and the other proportional to dU/dt which is referred to as the unsteady pressure term. The force in the direction of the flow, due to steady pressure, is known as the drag force (see Fig. 10-1). By integrating the steady pressure on the body surface one can determine the pressure drag force:

$$\begin{aligned} Drag \equiv F_D &= \int_{\theta=0}^{\theta=2\pi} p_{steady}(n_x)\ a\ d\theta \quad \text{(note: } n_x \text{ term is to find force along -ve x direction)} \\ &= \int_{\theta=0}^{\theta=2\pi} \frac{\rho}{2}U^2(1-4\sin^2\theta)(\cos\theta)\ ad\theta \quad \text{(note: } n_x = \cos\theta \text{)} \\ &= 0 \end{aligned}$$

Surprisingly some what, especially after all the work done to solve the potential flow problem, the results predict that the drag force on the cylinder is zero. In reality, because of viscosity, the flow would separate and thereby reduce the pressure on the aft portion of the cylinder. The difference of pressure between forward and aft portion of the cylinder gives rise to the form (pressure) drag (review Fluids I). The incorrect result of zero drag force predicted by the potential flow solution is referred to as the D'Alemberts Paradox.

Aside Material: A Review on Drag Force in Real Fluid: Before considering the force brought forth by the unsteady pressure terms, let us briefly review some important aspects of the drag force in a real fluid. The pressure coefficient defined as

$$C_p = \frac{p}{1/2\ \rho U^2}$$

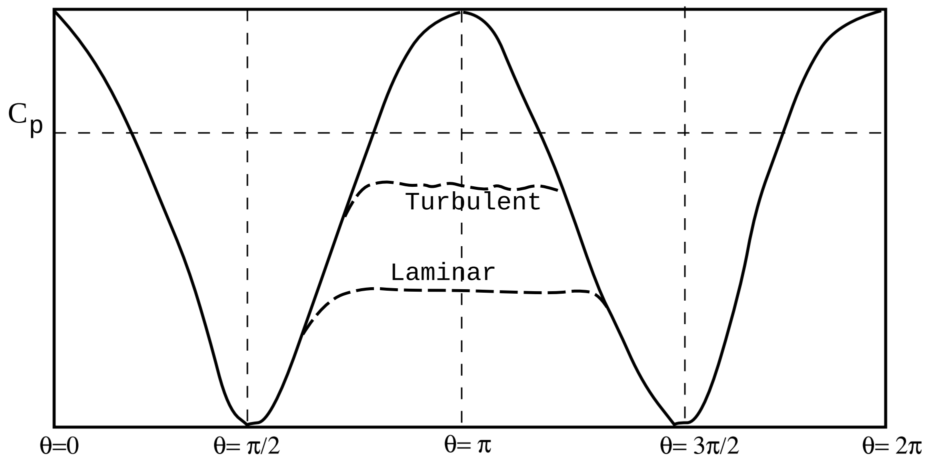

Fig 10-2. Pressure distribution on a cylinder: ideal flow vs. viscous flow

corresponding to steady pressure term, in the flow past a cylinder, is given by

$$C_p = 1 - 4\,\sin^2\theta$$

and it is plotted on Fig. 10-2.

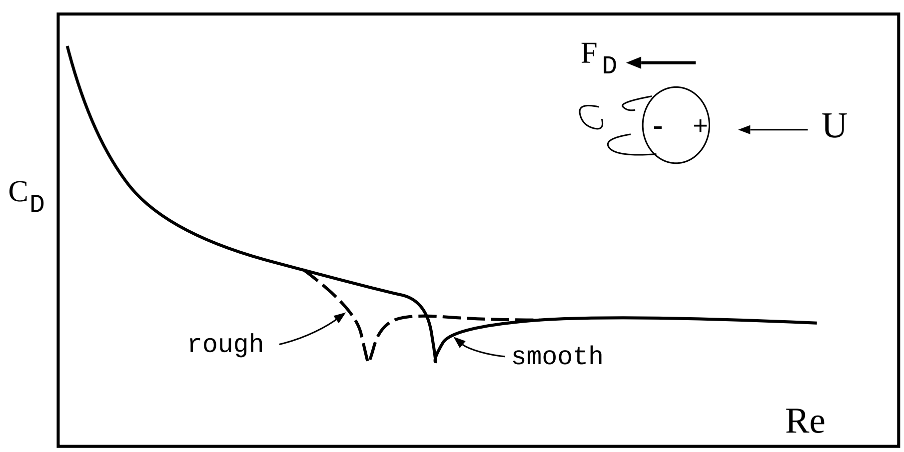

Fig 10-3. Drag coefficient vs. Reynolds Number of a Bluff Body

Note that, per potential flow results, the pressure coefficient symmetric above aft and forward portions of the cylinder (Fig. 10-2). Thus, one obtains zero drag in the case of potential flow. Also shown in the Fig. 10-2, in dotted lines, are the pressure coefficient curves corresponding to laminar and turbulent viscous flows past the cylinder. Note that the pressure drop in the case of turbulent flow is smaller than that of the laminar flow. This is because, turbulence delays (spatially speaking) separation and yields a narrower wake. Therefore, in the case of bluff bodies (bodies in which flow separtion occurs) turbulence reduces the drag coefficient. It is for that reason, for example, a golf ball is dimpled to induce turbulence. The drag coefficient curve of a typical bluff body is sketched in Fig. 10-3. Note that the "kink" in the curve, with a sudden drop of the drag coefficient, is due to the onset of turbulence. Also note that roughening the surface causes the turbulence to occur at a lower Reynolds number.

Roughening a surface is always not a good idea to lower the drag force. For example, in the case of streamlined bodies (bodies in which flow separation does not occur), the drag force is predominantly due to skin friction – ie. due to tangential or shear stress. The skin-friction coefficient is larger if the flow becomes turbulent. It is therefore essential to keep the surfaces smooth to ensure laminar flow and lower skin friction. A representative graph of skin-friction coefficient C_f of a thin streamlined body is shown in Fig. 10-4. Notice that C_f of the laminar flow is smaller than that of the turbulent flow and that the transition of the flow from laminar to turbulence depends on the surface smoothness.

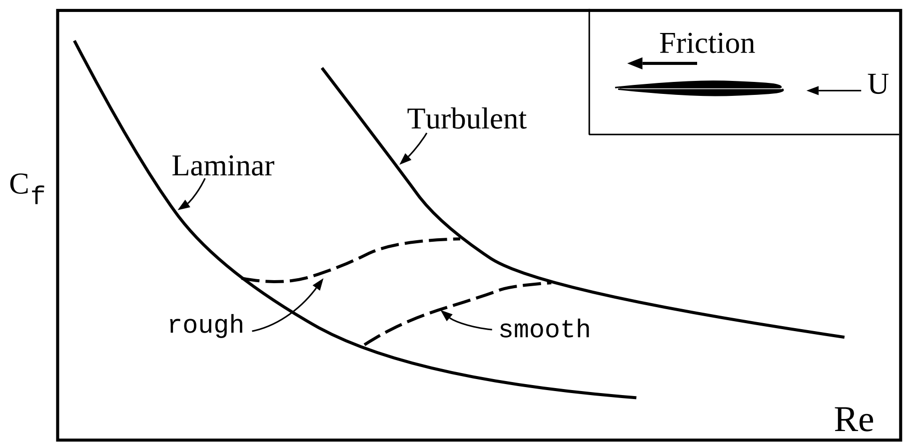

Fig 10-4. Skin-friction coefficient of a Streamlined Body

Final Remark: In the case of lifting surfaces (wings), the primary drag force component (prior to stall) is the induced drag which is a consequence of the trailing vortices. At and beyond stall, the pressure drag could become dominant. Inducing turbulence will delay separation and therefore stall and pressure drag. In the case of fast marine surface vehicles, the primary component of drag force is known as the wave resistance which is associated with the generation of surface waves by the vehicle. We will learn more about the induced drag of lifting surfaces and wave resistance of marine vehicles in Fluids II. Let us next return to the potential flow solution of flow past a cylinder and examine the force associated with the unsteady pressure term.