Lecture #9

VI. Inertia, Pressure-Grandient and Coriolis Effects

In the field measurements of inertia current reported in texts such as *Ocean Circulation* published by the Open University, one also notices a drift also. The trajectory of a purely inertial current is only a circle when observed from a rotating frame; the drift had to therefore have occured due to another mechanism. In this chapter, let us explore the effect of additional force of pressure-gradient on the current. In order to keep the resulting equations manageable, we will stick to the Lagrangian description of the flow and model the pressure-gradient force by a constant term. As we will see in the next chapter, the pressure-gradient forces can occur due to sea-surface slope and/or due to density startifications.

Allowing a constant-pressure gradient type force in the x (eastward) direction, the equations for inertia current are extended as follows:

$$\frac{du}{dt} = -K + fv \tag{1}$$

$$\frac{dv}{dt} = -fu \tag{2}$$

In the above, K a constant is a "model" for a positive pressure gradient toward east which will tend to accelerate the fluid in the negative x direction (westward), and hence $-K$ on the right-hand side of the first equation.

As in the analysis of inertia current, we shall assume

- The current is horizontal (ie., $w = 0$)
- f-plane approximation (ie., the Coriolis parameter is assumed constant)

Above represent a pair of coupled, first-order, ordinary differential equations for particle-velocity components u and v. Differentiating the Eqn.1 with respect to time and substituting Eqn.2 equation for the resulting dv/dt, we obtain

$$\frac{d^2u}{dt^2} = -f^2u \tag{3}$$

Upon solving the above second-order equation for u, one can determine v by using Eqn. 1:

$$v = \frac{1}{f}\{K + \frac{du}{dt}\} \tag{4}$$

Integrating Eqn.(3), we obtain

$$u = A \ \sin ft + B \ \cos ft \tag{5}$$

where A and B are integration constants. Without loss of generality, let us set $u = 0$ at time $t = 0$. With this initial condition, Eqn.(5) becomes

$$u = A \ \sin ft \tag{6}$$

Substituting Eqn.(6) in Eqn.(4), we obtain

$$v = A \ \cos ft + \frac{K}{f} \tag{7}$$

where the quantity K/f is a particular solution associated with the westward pressure-gradient force. The integration constant A can be determined from the magnitude of the current.

The above solution for u and v, due to combined effects of inertia, Coriolis and pressure-gradient forces

$$\frac{du}{dt} = -K + fv; \ \frac{dv}{dt} = -fu \ \rightarrow \ u = \ \sin ft; \ v = A \ \cos ft + \frac{K}{f}$$

sheds light on some interesting phenomena of oceanographic currents:

- Pressure-gradient force along the westward direction (high pressure on east, low pressure on west) results in a drift along north-south direction, and NOT in the westward direction! This may be counter-intuitive, but is a consequence of the Coriolis effect.
- In the northern hemisphere, where f is positive, the westward pressure-gradient force will result in northward drift.
- In the southern hemisphere, on the other hand, the westward pressure-gradient force will result in southward drift.
- The difference, in the direction of the current, depending on its occurrence in the northern or southern hemisphere is a Coriolis effect. We will discuss more such fascinating effects on Coriolis dominated flows in the following chapters.

Before moving to the next chapter on Geostrophic current, let us consider the direction of motion of a purely inertial current. Recall from the previous lecture, the equations for the trajectory (inertia circle) of an inertia current are given by

$$X - C_1 = -\frac{U}{f}\cos ft \quad (8)$$

$$Y - C_2 = +\frac{U}{f}\sin ft \quad (9)$$

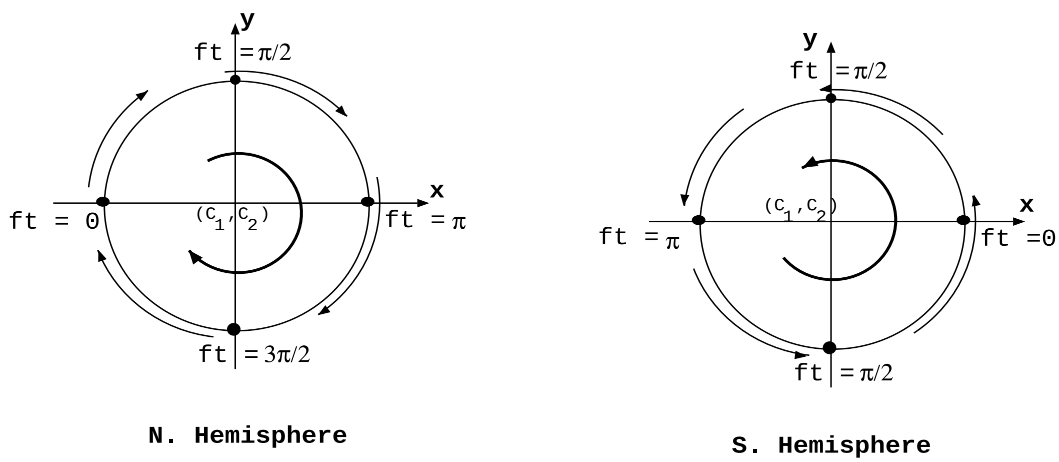

Fig 9-1. Motion of inertia current in Northern (left) and Southern (right) Hemispheres

When sketched, for positive f which is the case in the northern hemisphere, he trajectory will be clockwise. On the other hand, for negative f which is the case in the southern hemisphere, the above expressions become

$$X - C_1 = \frac{U}{|f|}\cos|f|t$$

$$Y - C_2 = +\frac{U}{|f|}\sin|f|t$$

(since t is positive, cos an even function and sin an odd function) the direction of motion is counterclockwise. To an observer in the northern hemisphere and facing the equator, the motion of the sun is clockwise; to an observer in the southern hemisphere and facing the equator, the motion of the sun is counter-clockwise. The direction of the inertia current is therefore referred to as *cum sole* (ie., with the sun!).