## I. Introduction

As the title suggests, the empahsis of the course will be on the physics of ocean flows, in particular, on various mechanisms driving ocean currents and waves. The mechanisms include wind, pressure gradients resulting from surface elevation and density gradients, thermal and pressure variations in the atmosphere.

### Descriptive .vs. Dynamical Oceanography

The subject of physical oceanography is treated by two somewhat different, and yet complementary, approaches namely *descriptive* or *synoptic* oceanography and *dynamical* oceanography. In the former, the ocean flows are studied based on direct observations and field data gathered by sailors and oceanographers over the years. A proper synopsis of the observations and data has revealed some important features and aspects of ocean flows. In the latter, *dynamical oceanography*, the subject is treated using fundamental principles of classical physics, mathematical modeling and analysis. Each of the two approaches has its limitations. For example, the predictive capability of the descriptive method is limited by the amount of data used to construct a particular model. The model may have to be revisited and revised as new information becomes available. The findings of descriptive oceanography, on the other hand, is limited by the assumptions and approximations used to construct a oceanographic model and in the analysis. The assumptions necessary, as we will see in due course of the course, in order to be able to tackle the governing equations of an oceanographic flow. In particular, oceanographic flows are random with disparate length and time scales and the governing equations highly nonlinear. Despite the differences in the approaches, the two could be complementary in that one could use descriptive aspects of oceanography to validate a dynamical model and, similarly, use a dynamical aspects of oceanography to synthesize data and observations. The focus of this course, in particular of the lectures, will be on the dynamical oceanography. Many topics in descriptive oceanography will be left as reading assignments.

## II. Equations Governing Oceanographic Flows

### Mathematical Formulation

In the dynamical oceanography, the governing equations are obtained using principles and laws of classical physics, such as

- conservation of mass which states that *mass is neither created nor destroyed*
- balance of momentum which is the same as the Newtons II Law which states that *the rate of change of linear momentum of a body is equal to the sum of the external forces acting on the body*
- work-energy theorem as given by the *First Law of Thermodynamics*
- Newton's law of gravitation which states that the force of attraction between two bodies, of masses $M$ and $m$ separated by a distance $R$, is proportional to the product of the masses and inversely proportional to the square of the separation distance; *ie.* $F = GMm/R^2$, where $G$ denotes the universal gravitational constant.
- etc,

mathematical identities and descriptions, such as

- Gauss theorems (or) integral identities which can be used to transform a volume integral into a surface integral (or *vice versa*). For example, let $\vec{A}$ denote a continuous vector field (function) and $a$ a continuous scalar field (function) defined in region $\forall$ bounded by surface $S$. Let $\hat{n}$ be unit normal vector on $S$ pointing outward of $\forall$. Then per Gauss theorem,

  $\int_\forall \nabla \cdot \vec{A} \; d\forall \equiv \int_S \vec{A} \cdot \hat{n} \; dS$

  $\int_\forall \nabla a \; d\forall \equiv \int_S a\hat{n} \; dS$

- Reynold's Transport theorem (Leibnitz theorem), as per which,

  $\frac{d}{dt} \int_{\forall(t)} f(x, y, z, t) \; d\forall \equiv \int_{\forall(t)} \frac{\partial f}{\partial t} \; d\forall \equiv + \int_S f \; V_n \; dS$, where $V_n$ denotes the normal velocity of the boundary surface $S$ of time-dependent volume $\forall$. In the case of so-called material volume (ie., that consisting of the same fluid particles), $V_n$ will be equal to the normal component of fluid velocity $\vec{u}$ on the boundary; *ie.*, $V_n = \vec{u} \cdot \hat{n}$ on $S$.

- Control-volume equation, as per which,

  $\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_\forall \; \rho\beta \; d\forall + \int_S \; \rho\beta\vec{u}_{rel} \cdot \hat{n} \; dS$ where $B$ denotes an *extensive* property, $\beta$ the associated intensive property, $\rho$ fluid density, and $\vec{u}_{rel} \cdot \hat{n}$ the normal velocity of fluid relative to boundary velocity.

- etc

- Lagrangian and Eulerian descriptions of flow, by which we mean particle and field descriptions of a flow, respectively. In classical physics, the particle (Lagrangian) description is commonly followed as the conservation laws can be applied in a straightforward manner. In fluid mechanics however, the Eulerian (field) description is commonly used. Both methods are correct, if properly formulated. The preference for one method over the other is purely a matter of convenience in a particular application. To give a very simplistic example, let us consider the problem of investigating the peak-hour traffic on I95 between West Palm Beach and Miami. We can do so by having a group of people drive down or up the highway and radio to the data-gathering center their locations and speeds at different times. This approach is referred to as the Lagrangian description, in which the independent variables are the locations of the people (particles) and time. One can also study the traffic, by recording the speed of the passing vehicles at various fixed stations such as Exit 1, Exit 2,...Exit 80 on I95 at various times. The second approach is called the Eulerian (field) description, in which the independent variables are fixed spatial points and time. The quantities measured will be described as field quantities; *eg.,* velocity field.

  Fluid particle acceleration in terms of particle velocity is simply,

$$\vec{a} = \lim_{\delta t \to 0} \frac{\vec{V}(t+\delta t) - \vec{V}(t)}{\delta t}$$

  where $\vec{V}$ denotes the particle velocity. In terms of velocity field $\vec{u}(x, y, z, t)$, the particle acceleration is given by the total time derivative of $\vec{u}$; in other words, the spatial variables are not fixed but correspond to the trajectory of the particle. Thus in terms of velocity

$$\vec{a} \equiv \frac{d}{dt}\vec{u}(x(t), y(t), z(t), t)$$

  which upon expansion becomes

$$\vec{a} = \frac{\partial u}{\partial t} + \frac{dx}{dt}\frac{\partial \vec{u}}{\partial x} + \frac{dy}{dt}\frac{\partial \vec{u}}{\partial y} + \frac{dz}{dt}\frac{\partial \vec{u}}{\partial z}$$

  where $(u, v, w)$ corresponds to $x, y, z$ components of $\vec{u}$. As $x(t), y(t), z(t)$ corresponds to fluid trajectory, $u = dx/dt$, $v = dy/dt$ and $w = dz/dt$. The expression for fluid particle acceleration in terms of velocity field can therefore be written as

$$\vec{a} = \frac{\partial u}{\partial t} + \left[u\frac{\partial \vec{u}}{\partial x} + v\frac{\partial \vec{u}}{\partial y} + w\frac{\partial \vec{u}}{\partial z}\right] = \frac{\partial u}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}$$

  The first term on the right is called the *local* acceleration and the second term the *convective* acceleration. Note that conservation laws of mechanics are defined with respect to particle acceleration (and not field acceleration $\frac{\partial u}{\partial t}$). The additional terms in the definition of particle acceleration in terms of velocity *field* is a price to be paid for the convenience of studying a flow using Eulerian description!

and various assumptions depending on the context, such as

- *continuum* hypothesis, in which the matter is assumed to be continuously distributed in space. In reality, at the *microscopic* or atomic level, matter is NOT continuously distributed but are discretly concentrated on particles that randomy move in time. In oceanography, or in any branch of fluid mechanics in general, the interest is on only *macroscopic* behavior of a flow. On the macroscopic level, one can define the medium to be a continuum. The continuum hypothesis allows the definition of density $\rho$ as mass per unit volume, or more precisely as
$$\rho = \lim_{\delta\Omega \to 0} \frac{\delta m}{\delta\Omega}$$
where $\delta m$ denotes the mass of matter in the volume $\delta\Omega$

- incompressible flow, in which particle density is assumed to be constant. It can be shown that in the case of a flow in which the flow velocity is very small compared to the speed of sound in the medium, the fluid can be assumed to be incompressible. Note that incompressibility and *homogeneity* mean different aspects of a fluid. A fluid is said to be homogeneous, if its density is same throughout the fluid. In the case of an incompressible fluid, on the other hand, different particles may have different densities but their densities remain constant over time as they move around because of the flow!

- inviscid flow, in which the viscosity effect is neglected.

- etc

to simplify the governing equations. The oceanographic flow problem will be defined using a rectilinear coordinate system fixed on the surface of the earth. Depending on the length and velocity scales of a flow, the earth-fixed coordinate system may be treated as inertial or rotational. For example, in flows such as that of the Gulf Stream, the effect of the earth's rotation – specifically the Coriolis force – is important and the coordinate system is treated as a rotating frame of reference. In flows such as surface gravity waves with wavelengths O(10 - 100 m), the Coriolis effect is negligible and therefore the earth-fixed coordinate system can be assumed to be an inertial frame.